

# Fractionalize This

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Precisely what are the electrons in a high-temperature superconductor doing before they superconduct? Strong electronic correlations may give rise to composite rather than fractionalized excitations, as is typical in other strongly coupled systems such as quark matter.

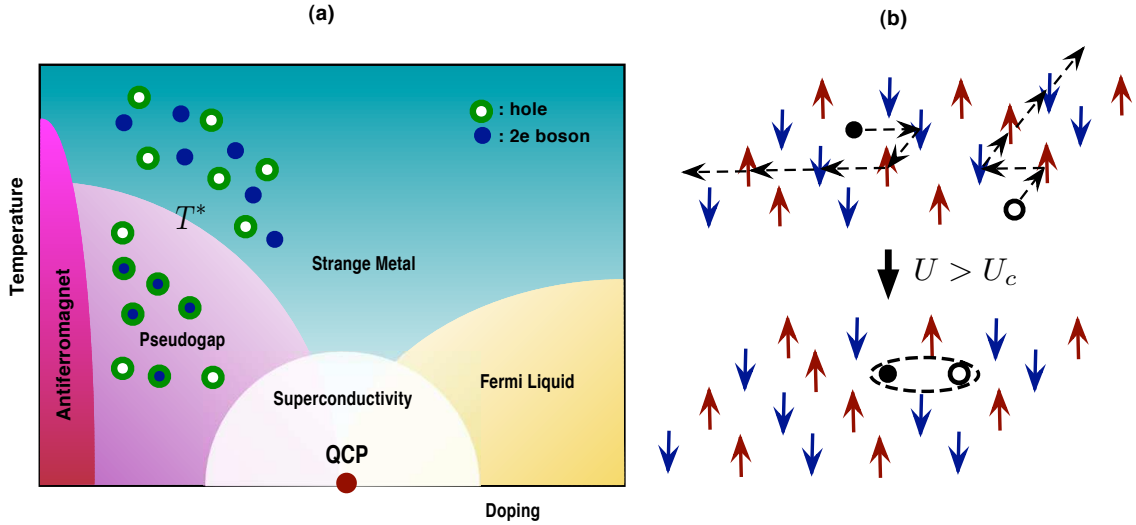


FIG. 1. Phase diagram and anomalous transport in the cuprate high-temperature superconductors. (a) Heuristic phase diagram as a function of holes doped into the copper-oxide plane. The pseudogap and strange metal are characterized by a depletion of the density of states and a  $T$ -linear resistivity, respectively. The pseudogap terminates at a zero-temperature critical point or quantum critical point (QCP). To the right is a Fermi liquid where weak-coupling accounts become valid. The picture advanced here for the pseudogap to strange-metal dichotomy is an unbinding transition involving charge  $2e$  bosons and holes. The charge  $2e$  boson is a collective excitation arising from the inseparability of the high and low energy scales in the cuprates. Above the pseudogap line,  $T^*$ , the charge  $2e$  boson and a hole unbind. Scattering of bosons and electrons above the temperature to create the boson naturally yields  $T$ -linear resistivity. (b) The mechanism for the insulating state in the parent material, a half-filled band, as proposed by Mott. Up (down) arrows indicate spin up (down) electrons. Below a critical value of the on-site interaction  $U$ , doubly occupied (solid circles) sites and holes (empty circles) are free to transport. Above a critical value of  $U$ , they are bound in localized pairs, thereby preventing conduction.

24 years after the discovery of superconductivity in the copper-oxide ceramics (hereafter cuprates), the central problem remains the anomalous properties of the normal state. The key anomaly is the strange metal (see Fig. (1)) in which the resistivity scales as a linear function of temperature rather than the characteristic  $T^2$  dependence of Landau's standard theory of metals. The fact that this transport anomaly persists to unusually high temperatures, roughly 1000K, indicates that it is a robust feature rather than some incipient nuisance that can just be dabbed away. As the phase diagram (see Fig. (1)) of the cuprates lays plain, the correct theory of the superconducting state should give rise to the panoply of phases that emerge at higher temperatures. That is, theories which describe only the superconducting state or just the pseudogap, a state with vanishing superconducting order but a gap in the single-particle spectrum nonetheless, are clearly inadequate. This suggests that the standard guiding principle of model building in which only  $T = 0$  states are relevant fails in this problem because neither the pseudogap nor the strange metal appear necessarily as zero-temperature phases in the cuprate phase diagram (at zero magnetic field). Instead, these phases emerge from the high-temperature correlated electron liquid, or charge vacuum, of the cuprates.

### Non-Fermi Liquid lost then found

Whatever this charge vacuum is, we know it must be a non-Fermi liquid. In modern terms, Landau's theory of a Fermi liquid rests on the key fact that all renormalizations arising from screened Coulomb interactions preserve the Fermi surface. Hence, as long as a Fermi surface exists and electrons are the charge carriers, the strength of the short-range repulsions is irrelevant. Consequently, any theory that claims  $T$ -linear resistivity from a perturbative analysis around a Fermi surface is moot. As a result, it is unfortunate that much of the current activity in the field of high-temperature superconductivity has focused on reclaiming a Fermi liquid picture in the region of the phase diagram precisely where it is least likely to apply. The primary impetus for this is the observation of coherent quantum oscillations indicative of closed orbits in high-magnetic field experiments[1] on a class of cuprates. Indeed, such experiments are central to the story at high magnetic fields. But that perhaps is it.

In this regard, the recent experiment by Fournier and co-workers[2] is important because it shows precisely how and where in the phase diagram the Fermi liquid picture fails. Any many-body problem is solved once the propagating degrees of freedom are isolated. These are the excitations which give rise to poles in the corresponding single-particle propagator, thereby describing coherent excitations. Fournier and colleagues[2] used angle-resolved photoemission spectroscopy (ARPES) to measure the strength of the pole in the electron propagator in  $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$  with truly

unprecedented accuracy and found it to vanish below a critical doping level of  $x \approx 0.15$ , indicating that electrons are not the propagating degrees of freedom at any momenta in the underdoped region. While this result is not particularly surprising given the series of experiments[3, 4] that preceded it, it comes at a crucial time when attention needs to be refocused on the strong coupling physics that is the high  $T_c$  problem.

### Limits of Quantum Criticality

Early in the theory of the cuprates, explanations[6, 9] based on some type of quantum criticality, that is, a phase transition at  $T = 0$  driven by quantum not classical fluctuations, were invoked to explain  $T$ -linear resistivity. The principle underlying such explanations is universality. Namely, at the quantum critical coupling, the only energy scale governing collisions between quasiparticle excitations of the order parameter is  $k_B T$ , where  $k_B$  is Boltzmann's constant. Consequently, the scattering rate between electrons scales as

$$\frac{1}{\tau_{tr}} \approx \frac{k_B T}{\hbar}, \quad (1)$$

with  $\hbar = h/2\pi$ ,  $h$  Planck's constant. Despite the fact that the momentum scales contributing to the scattering rate and the resistivity differ, a similar  $T$ -linear scaling was reasoned to hold for the resistivity as well[5]. But is this true? Quantum critical scenarios come in one of two types. Either the charge carriers are the critical modes or they are not but instead are coupled to a degree of freedom that is[7]. In the latter, it is well known[7] that the resultant resistivity scales as  $T^a$  where  $a$  is positive typically exceeding unity. What about theories in which the charges are critical? A few years ago, I showed with Claudio Chamon[8] that under three general assumptions, 1) the charges are critical, 2) charge is conserved, and 3) a single parameter governs the scaling of all energies and length scales, the dc resistivity must scale as

$$\rho(T) \propto T^{(2-d)/z}, \quad (2)$$

regardless of the underlying critical theory and the statistics of the particles. Here  $z$  is a number which measures how time and spatial increments are related. For example, if they are on the same footing,  $z = 1$ . For a three-dimensional ( $d = 3$ ) system such as the cuprates,  $T$ -linear resistivity obtains only when  $z = -1$ . This is an impossibility as it implies superluminal transport, thereby violating causality. Consequently, if the charges are critical, then either charge conservation must be abandoned, highly unlikely, or some new length scale (new degree of freedom) must emerge that is not governed by the same dynamics that underlies the divergence of the correlation length. Interestingly, breaking Fermi liquid theory in  $d = 2$  requires new degrees of freedom as well because simply cranking up the short-range interactions cannot destroy the Fermi surface.

### Mottness: Emergence of new degrees of freedom

Consequently, the central questions are what are the new degrees of freedom, how do they arise from the underlying microscopic theory of the cuprates, and how do they mediate  $T$ -linear resistivity? There is a highly successful account of  $T$ -linear resistivity known as marginal[5] Fermi liquid theory (MFLT). This approach is based on a phenomenological proposal for the self interaction of the electrons which explains 1) the broad spectral features seen in ARPES experiments[3, 4] at optimal doping and 2) also  $T$ -linear resistivity. While there have been recent attempts[9] to derive the key posit of MFLT from microscopics, no derivation exists that captures the strong interactions of the insulating state of the parent cuprates. This failure has caught the attention of the string theorists who have[10] brought to bear the gauge/gravity duality, which weds the geometry of space-time with quantum mechanics, on this problem. However, even this derivation[10] is phenomenological because the desired result is obtained by tuning the scaling dimension of a set of operators. It is the hope that in the quantum theory, the operators get quantized and hence acquire a definite scaling dimension. However, because the underlying quantum theory is not known, the precise physical content of the tuning parameter which gives rise to MFLT remains an open problem. Nonetheless, this approach demonstrates beautifully the inherent disconnect between a high-energy (UV) and a low-energy (IR) description of a strongly coupled system. To illustrate, the key claim of gauge/gravity duality is that some interacting quantum theories at strong coupling in  $d$ -space-time dimensions are dual to gravity theories in a  $d + 1$  dimensional space-time with a constant curvature, in this case one that is asymptotically anti-de Sitter ( $AdS_{d+1}$ ). In spite of the fact that the quantum theory resides at the boundary of the space-time, the  $UV$  scale, all the information regarding what the charges are doing at low frequencies is determined entirely by the near horizon metric  $AdS_2 \times \mathbb{R}^2$  (a space which is hyperbolic in two dimensions but flat in the other two), which is as far away from the boundary as possible. Hence, the degrees of freedom which emerge in the IR and govern the analytic structure of the theory have no correspondence with those in the original charged  $AdS_4$  UV limit. The same should also be true of the underlying microscopic quantum theory which ultimately describes marginal Fermi liquid theory. This suggests that extracting marginal Fermi liquid behaviour from the basic model of the cuprates might be tricky because the natural variables which would expose this behaviour are not the bare electrons at the UV. In fact, the operators whose scaling dimension gives rise to MFLT in the gauge/gravity theory have nothing to do with electrons. This conundrum has motivated any number of theories in which the electron is assumed to fractionalise. As is the case in other strongly coupled problems,

for example mesons (bound quark states) in quantum chromodynamics (QCD), the more likely outcome in the cuprate problem is that the excitations which emerge as being coherent are composite not fractionalised excitations.

Precisely what makes the charge vacuum in the cuprates unique was first uncloaked in a series of x-ray experiments by Chen, et al.[11] designed to measure the available phase space for adding a particle at low energies. They concluded that the available phase space is much larger than that expected for a Fermi liquid. Hence, it is important to understand this effect because it points to a clear source of the extra degrees of freedom. In a Fermi liquid, the number of available states for particle addition at low energy equals the number of holes created,  $x$ . Since each hole represents a quasiparticle, the quasiparticle weight scales as  $x$ . However, in the cuprates, the x-ray intensity at low energies increases faster than  $2x$ [11]. The basics of this effect are clear[12, 13] and intrinsically tied to the strong interactions in the parent state of the cuprates. Undoped the cuprates are essentially Mott insulators in that they possess a half-filled band but insulate nonetheless. They also order antiferromagnetically. However, that the x-ray intensity measured by Chen, et al.[11] increases faster than  $2x$ [11] follows solely from the strong interactions rather than from antiferromagnetic order. All such physics which is tethered to the strong interactions and not order is termed Mottness. Consider a simple model (due to Hubbard) for a Mott insulator in which electrons hop on a lattice with a matrix element of magnitude  $t$  but pay an energy cost  $U$  when they doubly occupy the same site. To understand the experiments of Chen, et al.[11], we need to consider how the spectrum rearranges upon the creation or removal of electrons, that is, hole doping. A hole leaves behind an empty site. Each such empty site can be occupied by either a spin-up or a spin-down electron. Hence, just from doping, the empty part of the spectrum at low energies has an intensity of  $2x$ . Further, since hole doping annihilates one state in the filled part of the spectrum, there are  $1 - x$  electron states (per site) remaining below the chemical potential. This gives rise to a total intensity of the lower band of  $1 + x$ . Based on this counting (and the Fermi liquid precedent that the quasiparticle weight should scale with the empty part of the spectrum), it is tempting to infer that the quasiparticle weight should scale as  $2x$  divided by the total weight of the low-energy band ( $Z = 2x/(1 + x)$ ) as has been proposed[14]. However, the Fournier, et al.[2] experiment shows that this scaling fails below  $x \approx 0.23$  with a vanishing weight at  $\approx 0.15$ .

How can the vanishing of  $Z$  be explained? Does the Chen, et al.[11] experiment help? Indeed it does because we have not accounted for the total intensity of the lower band by just counting electron states. The key point is that not all empty sites in the Hubbard model are created equally. Empty sites can be created simply by electron hopping. Consider an up-spin electron jumping to a site occupied by an electron with the opposite spin. Such empty sites automatically generate double occupancy and even exist in the insulating state as illustrated in Fig. (1b). The empty site left behind affects the excitation spectrum at all energies in that not only does the number of ways of creating doubly occupied sites decrease but so does the effective number of singly occupied sites[12, 15, 16]. In addition, the number of ways of adding a particle at low energies increases[15]. It has been known since 1967[16] that the total weight or intensity of the low-energy band, as a result of mixing with doubly occupied sites (termed dynamical spectral weight transfer), **increases** from  $1 + x$  (zero hopping limit) to  $1 + x + \alpha$ , where  $\alpha$  is a positive correction. The leading term in  $\alpha$  scales as  $t/U$  and as a consequence is generally ignored at strong coupling. However, retaining it leads to an important effect. (Recall even antiferromagnetism is a  $t/U$  effect which of course no one ignores.) While the intensity of the low-energy band increases when the hopping is turned on (or  $U$  is decreased), the number of ways electrons can be assigned still remains fixed at  $1 + x$ . Hence, dynamical spectral weight transfer poses a distinct problem: the total intensity of the low-energy band exceeds the number of electrons that can be assigned to this band. Such a mismatch cannot obtain in a Fermi liquid or band insulator because in such systems the spectral intensity of the bands is independent of the particle density. As a consequence, dynamical spectral weight transfer requires additional degrees of freedom that are not exhausted by counting electrons alone. Although not stated explicitly, this must have been ‘known’ to Harris and Lange[16] in 1967 because they refrained from dividing the spectrum into a filled and empty part based on the bare electron charge:  $1 - x$  filled and  $2x + \alpha > 2x$  empty states. If the total weight of a band exceeds the electron count, then what the electrons are doing is irrelevant! Likewise, if the phase space for adding a particle at low energies exceeds the number of ways electrons can be added to the band, then there are ways of adding a coherent excitation at low energies that are orthogonal to the addition of an electron. Hence, dynamical spectral weight transfer is directly linked to the vanishing of  $Z$ , thereby making it a property that affects ground and excited states alike. Since the intensity exceeding  $1 + x$  arises entirely in strong coupling, it should vanish once the doping level is such that weakly interacting physics obtains. This is the collapse of Mottness and is signaled by a decoupling of the upper and low-energy bands in a Mott insulator. Some experimental evidence for this has recently been seen by Lin, et al.[17] and in simulations by Jarrell and collaborators[18].

### **Doublon-holon unbinding: Strange Metal**

The relevant question then is: how can dynamical spectral weight transfer and the extra degree of freedom that describes it be captured by a low-energy theory of a doped mott insulator? The procedure put forth by Kenneth Wilson[19] for constructing a low-energy theory is to integrate out the unwanted high-energy stuff. In the context of the Hubbard model, it is typical to construct a low-energy theory by removing double occupancy. But as we have seen, such a procedure does more than Wilson would advise because without double occupancy, the dynamical correction

vanishes. While it is possible in projected schemes to retain the dynamical correction, isolating the dynamics it mediates is difficult because it is buried in cumbersome operator transforms[20]. Is it possible to represent the dynamical mixing with a new collective mode? Indeed it is. This new collective mode should have charge  $2e$  since it represents the mixing with double occupancy. The simplest procedure is to represent the physics of the upper band by an appropriately chosen coordinate rather than by particular electronic configurations. The subsequent Lagrangian[13] is quadratic in the coordinate for the high-energy scale and hence can be integrated out exactly to yield the low-energy physics. What results[13] is a theory with electrons and a new collective mode, a charge  $2e$  boson. The charge  $2e$  boson enters the theory initially as a Lagrange multiplier and hence is undamped. The low-energy physics it mediates typifies that of strong coupling. New charge  $e$  states ( in addition to the standard projected electron states in the lower band that give rise to the  $2x$  sum rule) emerge that have internal degrees of freedom and hence are orthogonal to a bare electron. The new states correspond to a bound state of the charge  $2e$  boson and a hole. At half-filling[13], the bound state (depicted in Fig. (1)) generates a charge gap and represents doublon-holon binding to which many previously have attributed (without proof) the Mott gap[21–23]. At finite doping, such binding persists yielding a pseudogap. Supporting this picture are recent oxygen K-edge experiments[17] in which the sign and magnitude of the temperature dependence of the dynamical contribution to the spectral weight across the pseudogap line is in excellent agreement with the prediction from the charge  $2e$  theory . The breakup of the boson-holon bound state generates the strange metal (see Fig. (1)). The mechanism for  $T$ –linear resistivity is simple within this model. Once the binding energy of the boson vanishes, bosons are free to scatter off the electrons. The absence of a kinetic energy term for the bosons implies that their dynamics are classical. The resistivity of electrons scattering off classical bosons is well-known to scale linearly with temperature above the energy to create the boson as depicted in Fig. (1). Hence, this mechanism is robust and should persist to high temperatures. Further, the presence of two distinct charge  $e$  states at low energy, one of which generates activated transport, naturally yields a two-fluid model which has been shown to underlie the temperature dependence of the Hall coefficient[24]. Consequently, the charge  $2e$  boson reduction of the Hubbard model offers a resolution of the pseudogap, the vanishing of the quasiparticle weight in the underdoped regime and the transition to the strange metal regime of the cuprates. The precise details of the unbinding transition (which preliminary work[25] indicates might be amenable to the gauge/gravity duality) and the role of the composite excitations in the superconducting state remain open.

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